Completing the square we have

$$x^2-x/3+1/4s^2=9-3/s+1/4s^2.$$

Hence $$x-1/2s=3-1/2s, \text{ or } x=3.$$

The fallacy will at once appear if it is observed that the same arguments could be used with respect to the two equations

$$x^2+y=9+\alpha \ldots (1),$$
$$y^2+x=3+\beta \ldots (2).$$

These may be written as follows:

$$x^2-9=\alpha-y=d \ldots (3),$$
$$x-3=\beta-y^2=sd \ldots (4).$$

As x cannot be equal to 3 for an arbitrary pair of values of α, β it follows that the method is fallacious. Just as in the preceding solution it is implicitly assumed that any point on $x^2-9=(x-2)/s$ must be common to the given parabolas. And the probability that this method should lead to the correct result in a given problem is again zero.

278. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

$xyz(\Sigma x)^2<3\Sigma y^2z\,\Sigma yz^2$, if x, y, z are positive.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{x^2y+y^2z+z^2x}{3}>(x^2y.y^2z.z^2x)^{1/3}>xyz. \quad \text{Also } \frac{xy^2+yz^2+zx^2}{3}>xyz.$$

$\therefore 3\Sigma y^2z\Sigma yz^2>27x^2y^2z^2>xyz.27xyz.$

$(x+y+z)^2$ is the greatest when $x=y=z$, and is then equal to $9x^2$.

$\therefore (x+y+z)^2<27xyz. \quad \therefore xyz[\Sigma(x)]^2<3\Sigma y^2z\Sigma yz^2.$

279. Proposed by THEODORE L. DE LAND, Treasury Department, Washington, D. C.

The United States Panama Canal Bonds were issued, to date August 1, 1906, and will mature on August 1, 1936; and they bear interest at the rate of 2% per annum, payable quarterly, on the first day of November, 1906, and the first day of February, May, and August, 1907, and so on for each succeeding quarter, until the bonds mature, when the principal will be paid at par with the last quarter's interest. The coupon bonds of this loan were quoted on the New York Stock Exchange, at 10.30 a. m., on December 17, 1906, at 103¾ bid and 104¾ asked.

Required: The rate of interest per annum, payable quarterly, an investor would *realize* if he purchased the Panama bonds on December 17, 1906, and could reinvest his interest income, quarterly, at the *realized* rate.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Let P=price of bond, n=number of quarter years to run, S=face of bond, x=realized rate of interest, r=rate of interest bond bears.

$P(1+\frac{1}{4}x)^n$=value of purchase money at the end of n quarter years.

$$\frac{Sr}{4}(1+\tfrac{1}{4}x)^{n-1}+\frac{Sr}{4}(1+\tfrac{1}{4}x)^{n-2}+\frac{Sr}{4}(1+\tfrac{1}{4}x)^{n-3}+\ldots+Sr+S$$

$$=S+\frac{\frac{Sr}{4}[(1+\frac{1}{4}x)^n-1]}{\frac{1}{4}x}$$

is the amount of money received on the bond.

$$\therefore P\tfrac{1}{4}x(1+\tfrac{1}{4}x)^n=\frac{Sx}{4}+\frac{Sr}{4}(1+\tfrac{1}{4}x)^n-\frac{Sr}{4}.$$

$$\therefore 104\tfrac{3}{4}(1+\tfrac{1}{4}x)^{119}x=100x+2(1+\tfrac{1}{4}x)^{119}-2,$$

$$(104\tfrac{3}{4}x-2)(1+\tfrac{1}{4}x)^{119}=100x-2,$$

$$\therefore x=.01794=1.794\%.$$

II. **Solution by the PROPOSER.**

The solution of the above problem requires that certain technical considerations be observed. The bond runs for 30 years, and as the interest is payable quarterly there are attached to it 120 coupons of \$0.50 each. The exact number of days in each quarter must be noted. The interest for the first quarter was paid November first. The next quarter (November, December, and January), contains 92 days, and interest has accrued from November first to include December sixteenth, or for 46 days; or the accrued interest is equal to $\frac{46}{92}$ of a quarter; or is equal to $\frac{1}{2}$ of \$0.50, or \$0.25, to the date of purchase. There is left to the date of the maturity $120-1.5$ or 118.5 quarters. The lowest price bid was \$103.75, and highest price asked was \$104.75. If there was a sale of these bonds on December 17 it is fair to assume that it was at either limit or between those limits, and we therefore assume that the sale was at the mean price or at \$104.25. To find the net investment we subtract the accrued interest from the mean price and have the net investment, or \$104.25−\$0.25, or \$104, the net investment.

Let $4X$=the realized rate of interest per annum, payable quarterly; or let X=the quarterly rate; let P=\$100=the face of the bond; n=the number of interest periods; I=the quarterly interest on \$100; and N=\$104=the net investment.

The amount of the net investment from the date of purchase to the date of maturity would be expressed symbolically by $N(1+X)^n$. The investor will at the date of maturity receive the face of the bond, P=\$100, and the amount of his quarterly interest considered as an annuity compounded at the rate X for the time n, which would be expressed, symbolically, by $I[(1+X)^n-1]/X$.

We can now equate the equivalent terms and obtain the following general equation:

$$N(1+X)^n = P + I[(1+X)^n - 1]/X \ldots (1).$$

By transformation and reduction (1) takes the following general form:

$$N = I/X - (I/X - P)/(1+X)^n \ldots (2).$$

Substituting numerical values we have:

$$\$104 = \$0.50/X - [\$0.50/X - \$100]/(1+X)^{118.5} \ldots (3).$$

We now have to find such a value of X that if it be substituted in (3) the two members will be equal. As the bond is sold at a premium we know that X must be less than 0.005. By trial we find it to be greater than 0.0045. We will first try 0.00456, and then 0.00457.

Take $X=0.00456$ in (2) and (3) and we have $I/X=\$0.50/0.00456$ $=\$109.649123$; from this amount take \$100 and we have \$9.649123. The logarithm of 9.649123 is 0.9844878. From this logarithm take 118.5 times the logarithm of 1.00456, or 0.2341442, and we have the logarithm 0.7503436; which is the logarithm of the number 5.627864; this number taken from 109.649123 gives 104.021259. This number is greater than the number in the first member of equation (3), or than 104, by 0.021259, which shows that the assumed value of X, or 0.00456, is too small.

Take $X=0.00457$ in (2) and (3) and we have $I/X=\$0.50/0.00457$ $=\$109.40919$; from this amount take \$100, and we have \$9.40919. The logarithm of 9.40919 is 0.9735522; from this logarithm take 118.5 times the logarithm of 1.00457, or 0.2346537, and we have the logarithm 0.7388985, which is the logarithm of the number 5.47149; this number taken from the number 109.40919 gives 103.93770. This number is less than the number in the first member of equation (3), or than 104, by 0.0623, which shows that the assumed value of X, or 0.00457, is too large.

Now as the assumed value, $X=0.00456$, gives a value too small, and the assumed value, $X=0.00457$, gives a value too large, the true value of X lies between these two assumed rates, the first assumption giving the nearer value. The approximate increase may be determined by Double Position:

$$\begin{array}{rcrcrl} 104.02126 & & 0.00457 & & 104.021259 & \\ 103.93770 & & 0.00456 & & 104.000000 & \\ \hline 0.08556 & : & 0.00001 & :: & 0.021259 & : \text{error} = e. \end{array}$$

Reducing the proportion we find $e=0.00000254$; add this to 0.00456 and we have 0.00456254 as a new trial value for X.

Try $X=0.0045625$ in equation (3) and we have a result approximately within a few mills of a true value.

Therefore the rate of interest per quarter which the investor will realize is $X=0.45625\%$; or $4X=1.825\%$, the investor's rate per annum, payable quarterly.

When there are three rates of interest to be considered in the problem, or the nominal rate expressed in the bond, the investor's rate, and the current or market rate, each per annum, payable quarterly, we let $4Y$=the current rate per annum, payable quarterly, then Y=the current quarterly rate. Put this value in equation (1) and we have:

$$N(1+X)^n=P+I[(1+Y)^n-1]/Y\ldots(4).$$

With equation (4) we have a perfectly general equation from which, by proper substitutions, we may apply it to any loan and find all values required, as N, P, I, n, X, and Y.

The logarithmic calculations in the above solution were made with Shortrede's tables of logarithms and antilogarithms. Should it be necessary to carry the decimals further it would be better to use ten-place tables.

Dr. Zerr's result differs from Mr. DeLand's in that he disregarded the technical consideration required in these bonds. The problem was also solved by A. H. Holmes. ED. F.

280 (Incorrectly numbered 180). Proposed by R. D. CARMICHAEL, Anniston, Ala.

Find values of x, y, z, and u satisfying the equations

$$x+y+z+u=10\ldots[1],$$
$$x^2+y^2+z^2+u^2=30\ldots[2],$$
$$x^3+y^3+z^3+u^3=100\ldots[3],$$
$$x^4+y^4+z^4+u^4=354\ldots[4].$$

Solution by E. A. ECKHARDT, 903 North Fifth Street, Philadelphia, Pa.

By multiplication, addition, and subtraction we obtain

$$\begin{array}{ll}
\Sigma xy=35 & \Sigma xy^3=x^3y+x^3z+x^3u+y^3x+y^3z+y^3u+z^3x+z^3y \\
\Sigma x^2y=300 & \qquad +z^3u+u^3x+u^3y+u^3z. \\
\Sigma xyz=50 & \quad =x^2(xy+xz+xu+yz+yu+zu) \\
\Sigma xy^3=646 & \qquad -x(xzu+xyu+xyz+yzu)+xyzu \\
\Sigma x^2y^2=273 & \qquad +y^3(x+u+z)+z^3(x+y+u)+u^3(x+y+z) \\
\Sigma x^2yz=404 & \quad =x^2\Sigma xy-4\Sigma xyz+xyzu+y^3(10-y) \\
xyzu=24 & \qquad +z^3(10-z)+u^3(10-u).
\end{array}$$

$$\Sigma xy^3=x^2\Sigma xy-4\Sigma xyz+xyzu+10(y^3+z^3+u^3)-(y^4+z^4+u^4).$$

Substituting, we have $646=35x^2-50x+24+10(100-x^3)-(354-x^4)$.

Whence, we have $x^4-10x^3+35x^2-50x=-24\ldots(2)$,

or $x^4-10x^3+35x^2-50x+24=0$.

The equations in y, z, and u are found to be identical to (2). The